

# Fourier Optics

## 1 Background

Ray optics is a convenient tool to determine imaging characteristics such as the location of the image and the image magnification. A complete description of the imaging system, however, requires the wave properties of light and associated processes like diffraction to be included. It is these processes that determine the resolution of optical devices, the image contrast, and the effect of spatial filters. One possible wave-optical treatment considers the Fourier spectrum (space of spatial frequencies) of the object and the transmission of the spectral components through the optical system. This is referred to as Fourier Optics. A perfect imaging system transmits all spectral frequencies equally. Due to the finite size of apertures (for example the finite diameter of the lens aperture) certain spectral components are attenuated resulting in a distortion of the image. Specially designed filters on the other hand can be inserted to modify the spectrum in a predefined manner to change certain image features.

In this lab you will learn the principles of spatial filtering to (i) improve the quality of laser beams, and to (ii) remove undesired structures from images. To prepare for the lab, you should review the wave-optical treatment of the imaging process (see [1], Chapters 6.3, 6.4, and 7.3).

## 2 Theory

### 2.1 Spatial Fourier Transform

Consider a two-dimensional object— a slide, for instance that has a field transmission function  $f(x, y)$ . This transmission function carries the information of the object. A (mathematically) equivalent description of this object in the Fourier space is based on the object's amplitude spectrum

$$F(u, v) = \frac{1}{(2\pi)^2} \int \int f(x, y) e^{i2\pi ux + i2\pi vy} dx dy, \quad (1)$$

where the Fourier coordinates  $(u, v)$  have units of inverse length and are called spatial frequencies.

Suppose a plane wave of amplitude  $E_0$  impinges on the object. The field distribution immediately behind the object  $E(x, y) = f(x, y)E_0$ , and the object information is impressed onto the light wave. There are optical processes that can produce the Fourier transform of  $E(x, y)$  and the object function, respectively.

### 2.2 Diffraction

When light (or any other wave) encounters an obstacle, it is diffracted. Diffracted waves are in general characterized by a complicated distribution of amplitude and phase. In certain cases, however, analytical expressions can be obtained for the diffracted field and the diffraction pattern that can be observed on a screen.

Consider a rectangular aperture characterized by the transmittance function  $f(x, y) = 1$  for  $-\frac{a}{2} < x < \frac{a}{2}$  and  $-\frac{b}{2} < y < \frac{b}{2}$  where the  $(x, y)$  plane is the plane of the aperture. It can be shown that the intensity distribution in the far-field (Fraunhofer) diffraction pattern is given by

$$I(u, v) = I_0 \frac{\sin^2(2\pi vb/2)}{(2\pi vb/2)^2} \frac{\sin^2(2\pi ua/2)}{(2\pi ua/2)^2} = I_0 \text{sinc}^2(2\pi vb/2) \text{sinc}^2(2\pi ua/2), \quad (2)$$

where  $u, v$  are the spatial frequencies for the  $x$  and  $y$  dimensions. For a point  $(x', y')$  in the observing plane at a distance of  $R_0'$  from the aperture center, the spatial frequencies are related to the sine of the angle between the normal to the aperture and the line from aperture center to  $(x', y')$  by

$$\begin{aligned} u &= -\frac{x'}{R_0'\lambda} \\ v &= -\frac{y'}{R_0'\lambda} \end{aligned}$$

Note that Eq. (2) is simply the absolute value squared of the Fourier transform of  $f(x, y)$ , i.e.  $|F(u, v)|^2$ . For small diffraction angles,  $R_0'$  can be identified with the distance between the diffracting and observation plane ( $\sin \alpha \approx \tan \alpha \approx \alpha$ ). In general, it can also be shown that the far-field or Fraunhofer diffraction pattern is the Fourier transform of the field across the diffracting aperture.

Similarly, diffraction from a circular aperture of diameter  $D$ , with a transmittance function  $f(r, \phi) = 1$  for  $0 \leq r < D/2$  and  $0 \leq \phi < 2\pi$ , yields

$$I(\rho) = I_0 \left[ \frac{2J_1(2\pi\rho(D/2))}{2\pi\rho(D/2)} \right]^2, \quad (3)$$

where  $J_1$  is the Bessel function of first order. The spatial frequency is  $\rho = \frac{\sin \theta}{\lambda} \approx \frac{r'}{R_0'\lambda}$  for small diffraction angles  $\theta$ . The other variables are:  $D$  = aperture diameter;  $r' = r_{\text{disk}}$  = radius of Airy disk in question;  $R_0'$  = distance from aperture to detection plane; and  $\lambda$  = wavelength. From this intensity pattern, one sees a dependence on the zeroes and extrema of the Bessel function. For example, the distance from the central intensity maxima to the first minimum,  $r_{\text{disk}}$ , gives the relation

$$1.22 = \frac{\pi D r_{\text{disk}}}{R_0'\lambda}, \quad (4)$$

while the distance from the central maximum to the next maxima gives

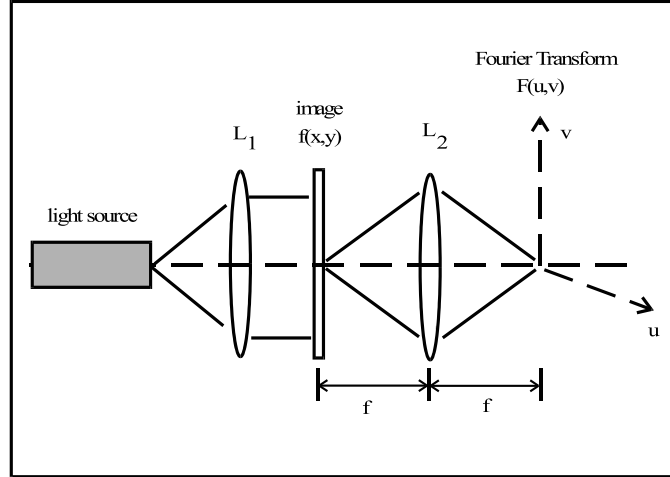
$$1.64 = \frac{\pi D r_{\text{disk}}}{R_0'\lambda}. \quad (5)$$

Strictly speaking the far-field diffraction pattern can only be observed at infinity. By placing a lens after the diffracting aperture the plane at infinity is imaged onto the focal plane of the lens. This explains why a lens can perform a Fourier transform.

### 2.3 Optical spatial filtering

**Fourier transform by a lens:** Optical spatial filtering is based on the Fourier transform property of a lens (see Fig. 1). It is possible to display the two-dimensional spatial frequency spectrum of an object in such a way that individual spatial frequencies can be filtered. This property is illustrated below. The object, in the form of a transparency, is illuminated by a collimated wave. The object is described by a space function  $f(x, y)$ . In the infinite lens limit, it can be shown [1] that the field distribution in the focal plane of the lens is given by

$$E(x_f, y_f) = \frac{iE_0}{\lambda f} \exp\{i[\omega t - k(S + f)]\} \exp\left[i\frac{kr_f^2}{2f^2}(S - f)\right] F(u, v). \quad (6)$$



**Figure 1:** Fourier Transform by a lens.  $L_1$  is the collimating lens,  $L_2$  is the Fourier transform lens,  $u$  and  $v$  are normalized coordinates in the transform plane.

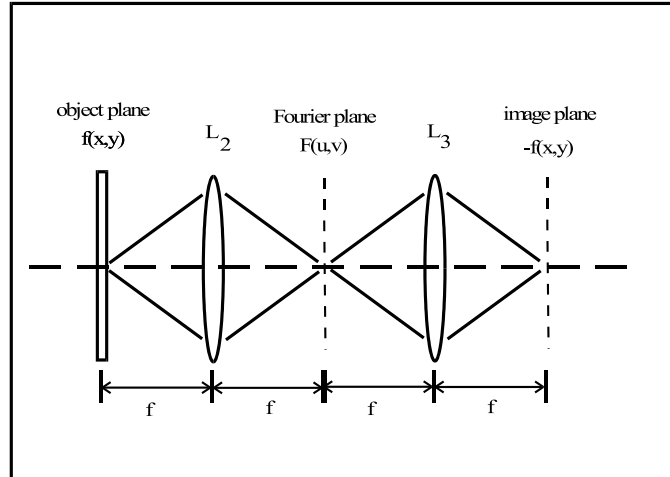
Here  $S$  is the object distance,  $f$  is the focal length of the lens,  $r_f^2 = x_f^2 + y_f^2$  are coordinates in the focal plane,  $F(u, v)$  is the Fourier transform of the object function,  $u = -x_f/\lambda f$ , and  $v = -y_f/\lambda f$ . Note, that the observable intensity pattern in the focal plane of the lens,  $|E(x_f, y_f)|^2$ , does not depend on the object position.

By making a Fourier transform of an image using a lens, it is possible to change the information in amplitude and phase that is supported by this image. For that purpose, a filter is placed in the spatial frequency plane  $(u, v)$ . A second lens, placed after the spatial frequency plane, is used to display the Fourier transform of  $F(u, v)T(u, v)$ , where  $T(u, v)$  describes the amplitude transmission of the filter. If the filter is removed, the output plane displays an inverted version of the input transparency (object).

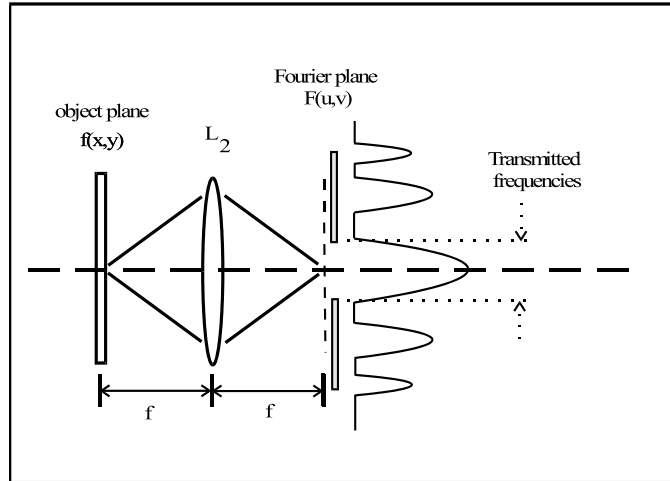
The basic optical processor is shown in Fig. 2. The object (a transparency) is illuminated by a coherent plane wave. Two identical lenses are used. Ray tracing shows that the system produces an inverted image of the object in the image plane. The first lens produces the Fourier transform of the object in its back focal plane. By manipulating the information in the Fourier transform plane, we obtain a “processed (filtered) image.”

**Spatial filtering:** The filtering consists of altering the image content by placing masks in the Fourier plane. These masks are to affect the amplitude and/or the phase of spectral components.

1. *Low pass filtering.* A simple kind of filter is the low pass filter frequently used in laser cavities to improve the beam quality. It allows low spatial frequencies to pass through the system while blocking components associated with higher spatial frequencies.
2. *High pass filter–edge enhancement.* Another important filter is the high pass filter (also known as an edge enhancer). A high pass filter consists of a dark spot in the center of the Fourier plane. A high pass filter is used to sharpen photographs. The high pass filter is sketched in Fig. 4.
3. *Image correction.* It is possible to do less trivial filtering. Consider for instance the photograph of a video display. It is possible to eliminate the lines from the picture by inserting two knife edges in the frequency plane. These two edges have to be put at a



**Figure 2:** Basic optical filtering system.



**Figure 3:** Low pass spatial filter

place where they cut the +1 and -1 diffraction orders of the grating formed by the lines of the screen. In this way, the picture is passed unchanged and the lines are eliminated.

4. *Image correlation.* More complicated filters can be made by photography. One interesting application is the comparison of various objects with a reference one, for example, a set of alphanumeric characters. We take one object, say 01, considered as a master. We record it on a photographic plate. Then we take another object and make its FT in the plane where the developed film is carefully placed. If the two objects are identical no light will pass through. If there is a small difference between the two objects, some light will reach the output plane. It is possible this way to test the reproducibility in the manufacturing of complex integrated circuits.

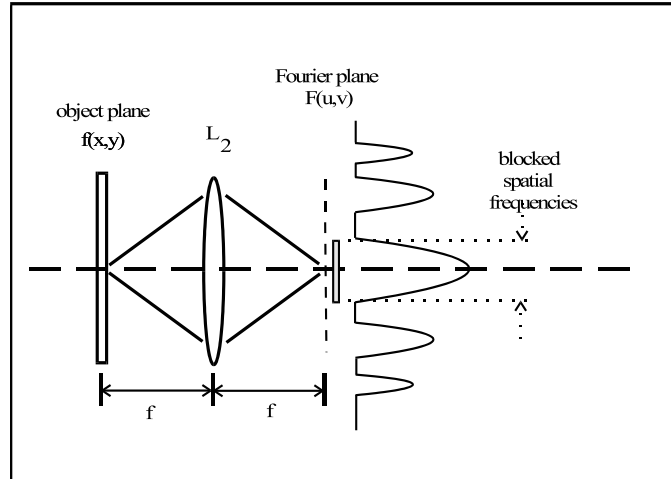


Figure 4: High pass spatial filter

## 3 Experiments

### 3.1 Tools and techniques

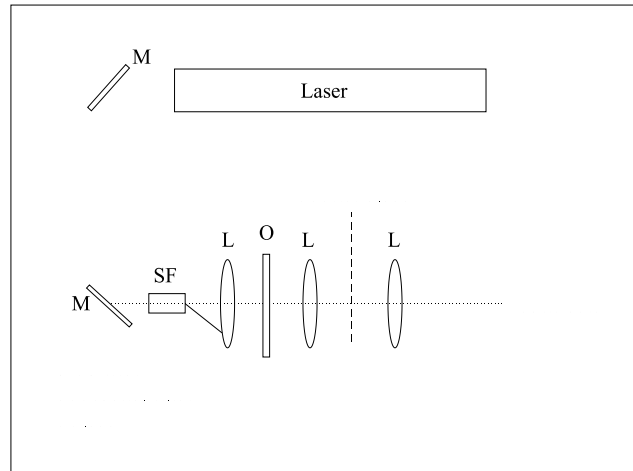
**Imaging with a CCD camera** To record images and spectra you will use a CCD camera (XC-75). Make sure that your beam is sufficiently attenuated to avoid saturation and damage. When you take images of the object slides you have to use an imaging setup that demagnifies. The detector chip of the camera has a size of  $7.95 \times 6.45 \text{ mm}^2$  (horizontal/vertical) with  $768 \times 494$  picture elements. The cell size is approximately  $8.4 \times 9.8 \mu\text{m}^2$  (horizontal/vertical), a typical object slide measures  $36 \times 24 \text{ mm}^2$ . Design an optical system BEFORE you attempt taking images of your object. Also, measure (calibrate) the actual pixel separation in horizontal and vertical direction using the translation stage. The actual procedure is up to your creativity.

**Imaging software programs** The camera is connected to a computer board and images can be acquired with a frame grabber program (WinTV2000). This allows you to capture images, display them on a computer, and save them as a bmp file for further processing. Do not forget to calibrate the image (size) of your camera.

To translate the gray tones to intensity values, to manipulate (filter) the images, and to display histograms of certain areas or lines, you need to use image processing software, for example ScionImage.

### 3.2 Getting the optical setup prepared

Begin by leveling the beam along the the optical rail (i.e. define an optic axis parallel to the rail). Align the spatial filter in order to get a clean expanded Gaussian beam. Make sure that all components of the bench are centered on the beam. Put the first lens in a location where it will transform the diverging beam from the spatial filter into a well collimated beam. Position the two Fourier lenses. The setup should look roughly like Fig. 5. You will need to modify the setup when you take images of your objects with a CCD camera.



**Figure 5:** Basic spatial filtering setup

### 3.3 Qualitative spatial filtering

Put a diffusing screen in the Fourier plane of the first lens and observe the diffraction patterns of various objects. Having recorded relevant observations, move the screen from the Fourier plane to beyond the last lens so that it is possible to examine the following and discuss the effects: low pass filtering; high pass filtering, any special filters (for example low pass horizontally but not vertically). Use the CCD camera to document some of your observations.

### 3.4 Quantitative spatial filtering

1. Select an object that shows parallel lines (amplitude square grating). Devise a procedure to determine the line width and separation.
2. Using the CCD camera, record an image of this object.
3. Record an image of the Fourier transform of this object.
4. With the information from 1., design a filter that allows you to transmit only those spectral components that are necessary to produce a cos grating that has half the original period in the image plane.
5. From the available set, choose an appropriate filter (that is, one that satisfies your design).
6. Place the camera so that you observe an image of the object. Insert your filter and record the filtered image. Discuss your observations. You should show a histogram of a line of both the original and filtered images in your report and base your discussion on these curves.

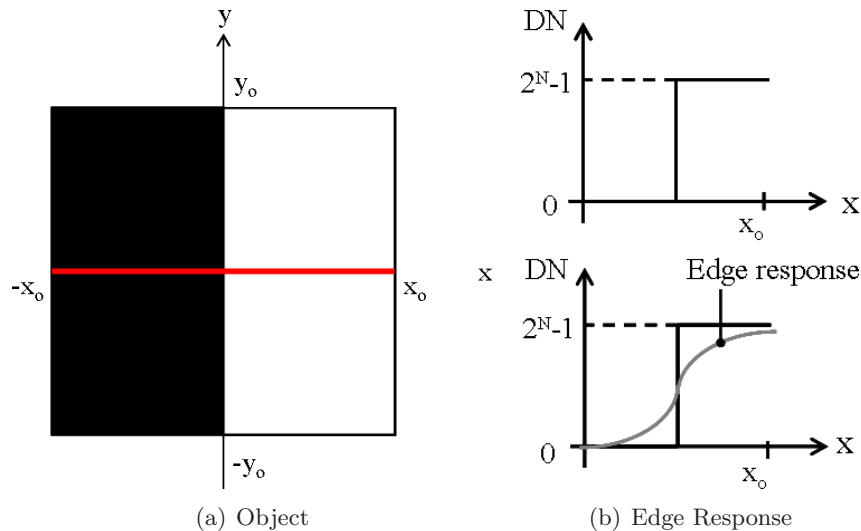
Follow a similar procedure to

1. Filter a more complex object. Select a slide that shows shapes with parallel lines superimposed (for example from a TV screen). Attempt to remove (some of) these lines.

- Clean up the beam profile of a laser. To this end, use the supplied slide and aperture down the beam diameter. This will result in the (ideal) reference beam. Mess up the beam by inserting a "dirty" slide. Design, produce and test a filter to improve the beam quality.

### 3.5 Spatial calibration

In this section, you will have the opportunity to perform a full spatial calibration of the imaging system. Following the same procedure of section 3.4 1-2, use an object with a single dark and white border feature. The image displayed by the CCD camera will look like the one in Fig. 6a. Using the software tool, draw a line across the border (like the red line in Fig. 6)a, and look at the profile along the x direction. Save the data for processing. This profile across the



**Figure 6:** Assessing the spatial blur of the imaging system on a target object (a) by comparing the ideal edge response (b TOP) to the actual response (b BOTTOM).

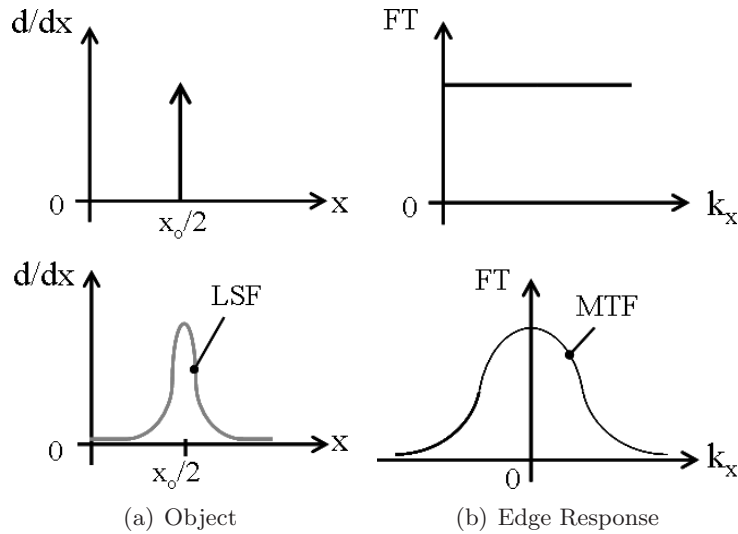
dark/white boundary, known as the “EDGE RESPONSE FUNCTION”, or ERF, of the system, is a signature of the amount of blur which is introduced by all the components of the system combined (lenses, apertures, electronics, etc...). Figure 6b shows a comparison between the ideal response (TOP) and the actual (BOTTOM) response of the system, with a line profile along X represented by digital numbers (DN) quantized at “N” bits. The derivative of the ERF, known as the “Line Spread Function”, or LSF, is the equivalent of the Point Spread Respond (PSF) of the system along a single dimensional. Ideally, the LSF would by a delta function (Fig. 7a TOP), but because of blur, it will display a characteristic as shown in Fig. 7a BOTTOM.

In this exercise, your task is:

- To measure the ERF along the x direction using a target object with the characteristics of the object in Fig. 6.
- From the saved data file, using a software such as Matlab or MAPLE (or any other that you are familiar with), find the LSF.
- Using the same software, find the system’s Modulation Transfer Function (MTF) along

the  $K_x$  direction. Remember that the MTF is the absolute value of the Fourier transform of the LSF.

- Assuming a circularly symmetric system, and neglecting edge effects, create an approximate 2-D MTF of the system and use it to calibrate the digital image of the object. Think about how a deconvolution of the 2-D PSF would enhance its contrast, only that, in this case, you are dealing in the spatial frequency domain.



**Figure 7:** Assessing the spatial blur of the imaging system on a target object by using information on its line spread function (a) and modulation transfer function.

## 4 Summary

1. *Qualitative spatial filtering.* Record observations for the Fourier transforms and the effect of filters for various objects and filter types.
2. *Quantitative spatial filtering.* Record images of a grating, of a complex object with superimposed lines, and of an ideal and messed-up laser beam. Design, print, and test Fourier filter. Note, that in order to maximize your lab time, much of the design work and preparation have to be done at home.
3. *Spatial calibration.* Perform spatial calibration of the imaging system. Measure the edge response across the black/white boundary of an object. Use recorded data to obtain an approximate 2-D Modulation transfer function of the system and use it to enhance the contrast of the digital image.



## References

- [1] Miles V. Klein and Thomas E. Furtak. *Optics*. John Wiley and Sons, ISBN 0-471-87297-0, New York, 1986.